

1) Consider the graph of a function f below.

a) [1] State the domain of f .

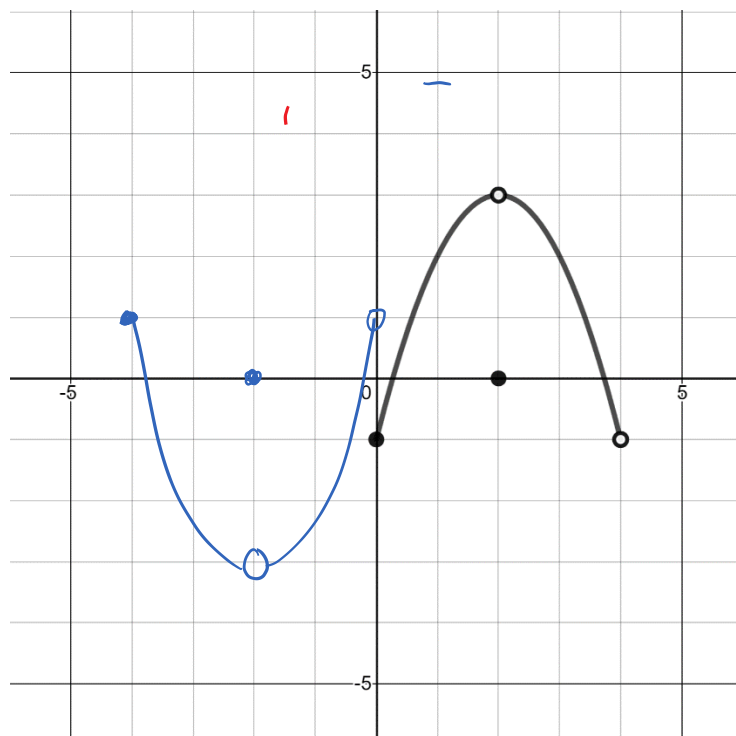
① $[0, 4]$

b) [1] State the range of f .

① $[-1, 3]$

c) [2] Sketch the graph of $g(x) = -f(x + 4)$.

① Vertical Reflection
① Shift 4 Left



2) [4] Suppose $f(x) = 3x - x^2$. Simplify the difference quotient: $\frac{f(4+h) - f(4)}{h}$, $h \neq 0$.

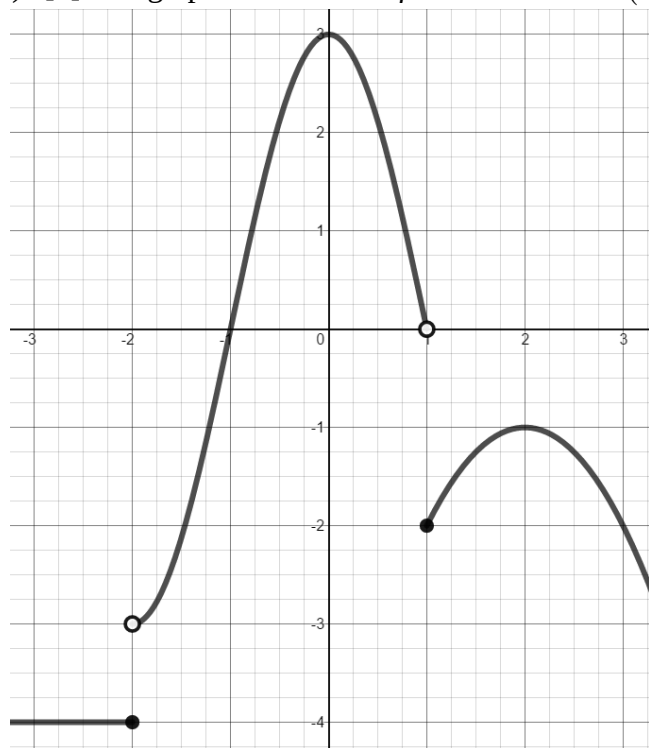
① $\frac{1}{h} [3(4+h) - (4+h)^2 - (-4)]$

① $= \frac{1}{h} [12 + 3h - (16 + 8h + h^2) + 4]$

① $= \frac{1}{h} h [-h - 5]$

① $= -h - 5$

3) [3] The graph of a function f with domain is $(-\infty, \infty)$ is given. Fill in the box to complete the statements.



a) As $x \rightarrow -\infty$, $f(x) \rightarrow$

-4

b) As $x \rightarrow -2^+$, $f(x) \rightarrow$

-3

c) As $x \rightarrow \infty$, $f(x) \rightarrow$

$-\infty$

4) [3] Find the standard equation of the quadratic function whose graph has vertex $V(-1, -2)$ and passes through the point $P(2, 3)$.

$$\textcircled{1} f(x) = a(x-h)^2 + k$$

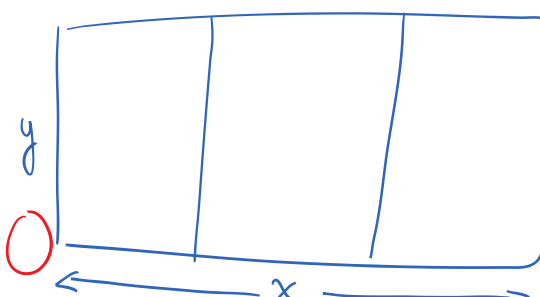
$$\textcircled{1} = a(x+1)^2 - 2$$

$$3 = a(3)^2 - 2$$

$$a = \frac{5}{9}$$

$$\textcircled{1} \therefore f(x) = \frac{5}{9}(x+1)^2 - 2$$

- 5) [4] A farmer wishes to put a fence around a rectangular field and then divide the field into three rectangular plots by placing two fences parallel to one of the sides. If the farmer can afford only 2000 meters of fencing, what is the maximum rectangular area?



① $A = xy = -2y^2 + 1000y$

① $y = -\frac{b}{2a} = -\frac{1000}{2(-2)} = 250$

$\therefore x = 500$

① $2x + 4y = 2000$

$x + 2y = 1000$

$x = 1000 - 2y$

$A_{\max} = 500(250)$

① $= 125,000 \text{ m}^2$

- 6) [3] Find a composite function form $y = f(u)$, $u = g(v)$, and $v = h(x)$ where $y = \frac{2}{(x^2 - 3x + 7)^3}$. Use nonidentity functions.

Answers may vary

① $f(u) = \frac{2}{u}$

① $g(v) = v^3$

① $h(x) = x^2 - 3x + 7$

- 7) [2] Several values of two functions T and S are listed in the tables below.

If possible, find

① a) $(T \circ S)(3) = T(S(3)) = T(2) = 1$

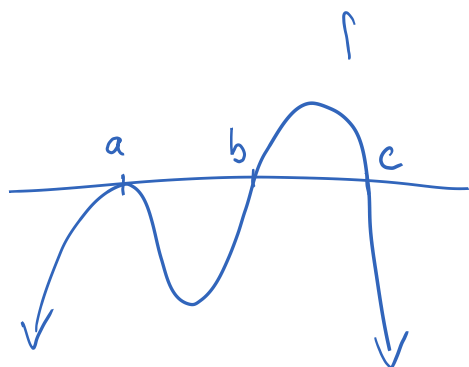
t	0	1	2	3	4
$T(t)$	2	3	1	0	5

① b) $(S \circ T)(4) = S(T(4)) = S(5) = \text{not defined}$

x	0	1	2	3	4
$S(x)$	1	0	3	2	5

8) Consider $f(x) = (x-a)^2(b-x)(x-c)$ where $a < b < c$. ^{note}

a) [3] Sketch a rough graph of f . Ensure you show the x -intercepts as well as the end behavior.



① End Behaviour

① x -intercepts

① "Bounce" at a

b) [2] Solve $f(x) \geq 0$. $\{a\} \cup [b, c]$

①

①

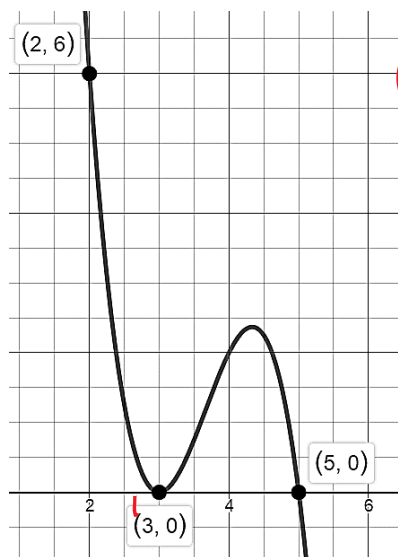
9) [3] Use synthetic division to $f(-2)$ where $f(x) = -x^3 + x + 5$.

② {

$$\begin{array}{r|rrrr} -2 & -1 & 0 & 1 & 5 \\ & 2 & -4 & 6 & \\ \hline & -1 & 2 & -3 & 11 \end{array}$$

① $\therefore f(-2) = 11$

10) [3] Find the formula of a polynomial function of degree three whose graph is shown below.



$$\textcircled{2} f(x) = a(x-3)^2(x-5)$$

$$6 = a(-1)^2(-3)$$

$$a = -2$$

$$\textcircled{1} \therefore f(x) = -2(x-3)^2(x-5)$$

11) [5] The graph of a rational function f is shown. Find a formula for f that satisfies all the conditions listed.

$\textcircled{1}$ - Vertical asymptote of $x = -1$

$\textcircled{1}$ - Horizontal asymptote of $y = 0$

$\textcircled{2}$ - Hole at $P(1,2)$ - common factor
- find a

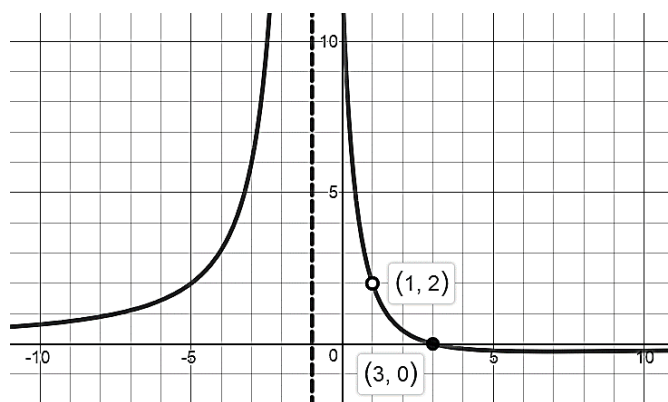
$\textcircled{1}$ - X-intercept of $(3,0)$

$$f(x) = \frac{a(x-3)(x-1)}{(x+1)^2(x-1)}$$

$$2 = \frac{a(-2)}{2^2}$$

$$a = -4$$

$$\therefore f(x) = \frac{-4(x-3)(x-1)}{(x+1)^2(x-1)}$$



12)[3] Find the slant asymptote for the graph of $f(x) = \frac{x^3 - 3x^2 + 6}{x^2 + 2x + 4}$.

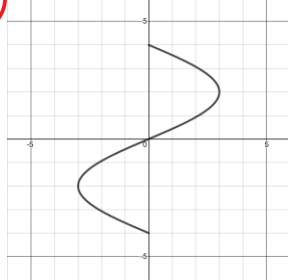
(2)
$$\begin{array}{r} x - 5 \\ x^2 + 2x + 4 \overline{) x^3 - 3x^2 + 0x + 6} \\ \underline{-(x^3 + 2x^2 + 4x)} \\ -5x^2 - 4x + 6 \\ \underline{-(-5x^2 - 10x - 20)} \\ 6x + 26 \end{array}$$

(1) S.A. is $y = x - 5$

13)[1] If the point $(5, -6)$ is on the graph of a one-to-one function f , find a point on the graph of f^{-1} .

(1) $(-6, 5)$

(1) 14)[1] Does the graph represent a one-to-one function? Explain why or why not.



No, fails vertical line test

15)[3] Find a formula for f^{-1} where $f(x) = \frac{7}{3-x}$

(1) $x = \frac{7}{3-y}$

(1) $3-y = \frac{7}{x}$

(1) $\therefore f^{-1}(x) = 3 - \frac{7}{x} \text{ OR } \frac{3x-7}{x}$